The optimal length of a rifle barrel

Pirooz Mohazzabi and Brad M. Shefchik

Abstract: A simple thermodynamic model for projectile propulsion inside a rifle barrel results in a functional relationship between the muzzle speed of the rifle and its barrel length. This relationship is in excellent agreement with the existing experimental data over a wide range of barrel lengths.

PACS Nos.: 45.40.Gj, 89.00

Résumé : Un modèle thermodynamique simple pour décrire la propulsion d'un projectile à l'intérieur d'un canon de carabine mène à une relation fonctionnelle entre la vitesse à la bouche et la longueur du canon. Cette relation est en excellent accord avec les données expérimentales disponibles pour une vaste gamme de longueurs de canon.

[Traduit par la Rédaction]

1. Introduction

Until recently, many shooters have been under the misconception that the longer a rifle barrel the higher the muzzle speed. This is obviously untrue, since the expansion of the gaseous products of the propellants can only proceed to a certain level before the friction between the bullet and the internal wall of the barrel dominates. Recently, experimental muzzle speeds as a function of barrel length have been reported in the literature for different types of 0.22 rimfire ammunition and over wide ranges of barrel lengths [1,2]. These results, which are shown in Table 1, indicate that initially the muzzle speed increases with barrel length but eventually tends to become a maximum for some optimal length of the barrel. There is, however, some uncertainty as to the exact position of this maximum because, in all four cases, only the last data show a small decrease in the muzzle speed, which could be within the limits of the experimental error.

Motivated by the tempting question regarding the relationship between barrel length and muzzle speed, the invention of the Big Berthas and the Paris Guns by Germans during World War I [3], and intrigued by the aforementioned experimental data on the 0.22 rimfire ammunition, we decided to obtain a theoretical relationship between muzzle speed and barrel length of a rifle based on the thermodynamics of the barrel interior.

2. Chemistry and thermodynamics of barrel interior

Gun powders used as propellants are generally chemical compounds or mixtures of chemical compounds that produce large volumes of gases at controlled, predetermined rates upon decomposition or deflagration inside the rifle or gun barrel [4]. Most of the modern propellants produced today are

Received 27 April 2001. Accepted 20 October 2001. Published on the NRC Research Press Web site at `http://cjp.nrc.ca/` on 1 May 2002.

P. Mohazzabi[1] and B.M. Shefchik. Department of Physics, University of Wisconsin-Parkside, Kenosha, WI 53141, U.S.A.

[1]Corresponding author (e-mail: pirooz.mohazzabi@uwp.edu).

 DOI: 10.1139/P01-147

Table 1. Measured muzzle speeds as a function of barrel length for four different types of 0.22 rimfire ammunition.

	Muzzle speed (ft/s)			
Barrel length (in.)	CCI Stinger	Win Wildcat	Rem HV	Fed UM*
2	948	794	791	
4	1236	1004	1020	
6	1374	1088	1092	1057
8	1441	1152	1104	
10	1464	1176	1150	
12	1502	1198	1167	1126
14	1531	1212	1180	
16	1539	1232	1194	
18				1145
18.5	1536	1230	1190	
24				1138

*UltraMatch (1000A).

single-based, double-based, or multi-based. Single-based propellants consist of nitrocellulose alone, $[C_6H_7O_2(ONO_2)_3]_n$, known as *guncotton*, with much the same appearance as ordinary cotton [5]. Double-based propellants consist of nitrocellulose and nitroglycerin, $C_3H_5(ONO_2)_3$, as a liquid explosive plasticizer [4, 6]. Multi-based propellants incorporate a crystalline explosive such as nitroguanidine in the mixture in addition to nitrocellulose and nitroglycerin. Finally, small amounts of additives may be used to facilitate handling, improve ignitability, and decrease muzzle flash [4]. The general goal is to produce a propellant that decomposes into gas slowly at the beginning and more rapidly as the decomposition progresses [6].

Upon deflagration, propellants produce a mixture of highly compressed gases at a very high temperature. These gases, depending on the exact chemical composition of the propellant used, are generally a mixture of the diatomic and triatomic molecules CO, CO_2, H_2O, N_2, O_2, and H_2. The pressure and the temperature of the products can be calculated from the enthalpies of the decomposition reactions. Peak pressures on the order of 3700 atm are not uncommon in the barrel where temperatures can be as high as 3000°C [7]. Although this temperature far exceeds the melting point of the steel from which the barrel is made, it only lasts for a very short period of time, causing a small amount of wear. The high-temperature, high-pressure gases expand to propel the projectile inside the barrel of the gun or rifle. Many computer programs have been developed to numerically predict the performance of these systems [8].

Since the temperature of the gaseous products in the rifle barrel is high, to a good approximation, the mixture of the gases can be treated as an ideal gas. Furthermore, since the expansion of the gases during the projectile propulsion takes place extremely fast, there would be no time for heat to exchange between the gas and the surroundings, including the metal of the barrel. Consequently, after complete decomposition of the propellant, the expansion of the gases takes place adiabatically, for which we have [9]

$$pV^{\gamma} = \text{constant} \tag{1}$$

where γ is the ratio of the molar heat capacity of the gas at constant pressure to that at constant volume

$$\gamma = \frac{C_\mathrm{p}}{C_\mathrm{V}} \tag{2}$$

For a general ideal gas, the molar heat capacities at constant pressure and constant volume are related by [10]

$$C_{\mathrm{p}} = C_{\mathrm{V}} + R \tag{3}$$

where R is the ideal-gas constant. The molar heat capacity at constant volume, in turn, is given by [11]

$$C_{\mathrm{V}} = \frac{1}{2}(3 + n_{\mathrm{rot}} + 2n_{\mathrm{vib}})R \tag{4}$$

where n_{rot} and n_{vib} are the rotational and vibrational degrees of freedom of the molecules, respectively. Equations (2)–(4), therefore, reduce to

$$\gamma = 1 + \frac{2}{3 + n_{\mathrm{rot}} + 2n_{\mathrm{vib}}} \tag{5}$$

As an example, for a fully excited water molecule where $n_{\mathrm{rot}} = 3$ and $n_{\mathrm{vib}} = 3$, (5) gives $\gamma = 1.17$. At 1500 K, on the other hand, calculations of the vibrational partition function for water show that most of the molecules are still in their vibrational ground state [12], giving a value of approximately 1.33 for γ. Finally, at much lower temperatures, where the molecules translate but do not rotate or vibrate, we get $\gamma = 1.67$.

The values of γ are listed in the literature for mixtures of gaseous products of various propellants [13]. They all fall within the range of 1.21 to 1.26, with an average value of 1.235 ± 0.025. This means that at the temperatures inside the barrel the rotational energy levels of the molecules are fully excited and the vibrational levels are partially excited. We shall use a value of $\gamma = 1.235$ for our model in the remainder of the article.

3. The model

The propellant generally does not detonate or explode upon ignition. It burns at a controlled rate, depending on the particle size and chemical composition. This controlled burning prevents damage to the barrel and other components of the rifle or gun, and at the same time, it increases the impulse delivered to the bullet inside the barrel.

Although accurate burning rates can only be obtained experimentally, various equations have been proposed for the rate of deflagration of propellants. For example, for propellants burning at high gun pressures [14]

$$r = a_1 + a_2 p \tag{6}$$

where r is the linear burning rate, p is the pressure, and a_1 and a_2 are constants. Since r is proportional to the rate at which the pressure increases, we have

$$\frac{\mathrm{d}p}{\mathrm{d}t} = b_1 + b_2 p \tag{7}$$

where b_1 and b_2 are new constants. With the initial condition $p(0) = 0$, this equation integrates to

$$p = \frac{b_1}{b_2}(\mathrm{e}^{b_2 t} - 1) \tag{8}$$

and the pressure increases exponentially with time during the burning process.

The burning stage, although considered slow in terms of the time scale involved in the firing process, is completed in a time of 0.5 to 1 ms. During this time, the bullet will move a distance ranging from 1.3 cm (0.5 in.) to 3.8 cm (1.5 in.) in most center-fire rifles and somewhat less in most handguns. Also, the deflagration of the propellant is known to be completed in the cartridge case, not the barrel [15]. There is no reason why these characteristics should be any different in the case of rimfire rifles and handguns.

Consider a reference volume V_1 inside the barrel in which, or perhaps in a smaller volume, the deflagration of the propellant is already completed. This reference volume does not necessarily have to be the initial volume during the projectile propulsion. The gaseous products produced in the barrel expand very rapidly, in a time on the order of a millisecond, from V_1 to a final volume V, corresponding to the point where the bullet emerges from the muzzle. Since the expansion of the gaseous products takes place adiabatically, as discussed earlier, we have

$$pV^{\gamma} = p_1 V_1^{\gamma} \tag{9}$$

where p and p_1 are the pressures corresponding to the volumes V and V_1, respectively.

As the bullet is pushed down the barrel by the expanding gases, it experiences a frictional force by the bore. Let the mean value of this frictional force be f. Therefore, the net work done on the bullet during its travel down the barrel is given by

$$W_{\text{net}} = \int_{V_1}^{V} p\,\mathrm{d}V - f(l - l_1) \tag{10}$$

where the first term is the work done by the adiabatic expansion of the gas. The second term is the work done on the bullet by the force of friction, where l and l_1 are the lengths of the barrel corresponding to the volumes V and V_1, respectively. Substituting from (9) and integrating, we obtain

$$W_{\text{net}} = \frac{p_1 V_1^{\gamma}}{\gamma - 1}\left(V_1^{-\gamma+1} - V^{-\gamma+1}\right) - f(l - l_1) = \frac{p_1 V_1}{\gamma - 1}\left[1 - \left(\frac{V}{V_1}\right)^{-\gamma+1}\right] - f(l - l_1) \tag{11}$$

which further reduces to

$$W_{\text{net}} = \frac{p_1 l_1 A}{\gamma - 1}\left[1 - \left(\frac{l}{l_1}\right)^{-\gamma+1}\right] - f(l - l_1) \tag{12}$$

where A is the internal cross-sectional area of the barrel.

According to the work–energy principle, the net work done on the bullet is equal to the change of its kinetic energy, thus

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2 = \frac{p_1 l_1 A}{\gamma - 1}\left[1 - \left(\frac{l}{l_1}\right)^{-\gamma+1}\right] - f(l - l_1) \tag{13}$$

where the first term on the left side is the kinetic energy of the bullet at the barrel length l, which we take to be the muzzle, and the second term is that at the reference length l_1. A few simple algebraic manipulations reduces (13) to

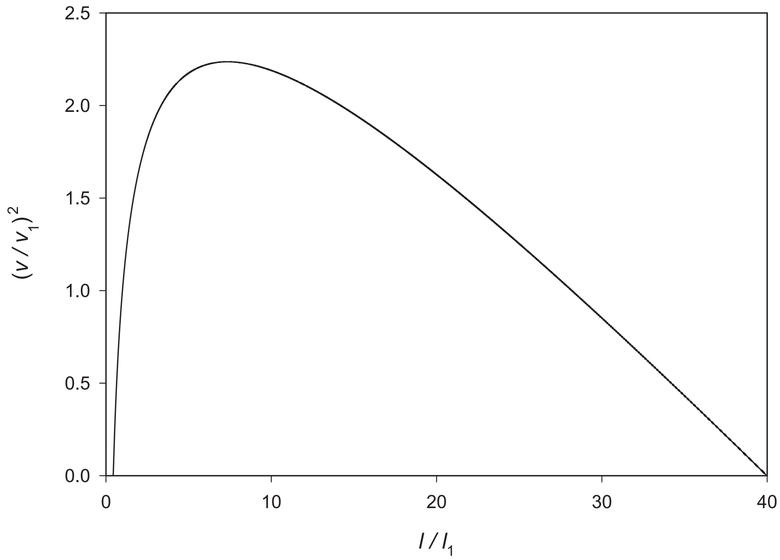
$$y = a(1 - x^{1-\gamma}) - b(x - 1) + 1 \tag{14}$$

where the dimensionless quantities x, y, a, and b are defined by

$$x = \frac{l}{l_1}, \qquad y = \frac{v^2}{v_1^2}, \qquad a = \frac{2p_1 l_1 A}{mv_1^2(\gamma - 1)}, \qquad b = \frac{2fl_1}{mv_1^2} \tag{15}$$

Note that y is also the ratio of the kinetic energy of the projectile at l to that at l_1.

Fig. 1. A graph of the function given by (14), describing the relationship between muzzle speed v of a projectile and barrel length l. v_1 is the speed of the projectile inside the barrel at the reference length l_1.



Equation (14) describes the variation of the muzzle kinetic energy and hence muzzle speed as a function of the length of the barrel for a rifle or gun. Figure 1 shows a graph of this equation with $\gamma = 1.235$, $a = 5$, and $b = 0.1$. As can be seen from the graph, the muzzle speed first increases, becomes a maximum, and then decreases to zero as the expanding gases become depleted of enough pressure to deliver momentum to the bullet and overcome the frictional force of the bore.

Two interesting features of (14) are worth noting. First, the maximum of the function y occurs at

$$x_{\max} = \left[\frac{a(\gamma-1)}{b}\right]^{1/\gamma} = \left(\frac{p_1 A}{f}\right)^{1/\gamma} = \left(\frac{F_1}{f}\right)^{1/\gamma} \tag{16}$$

where F_1 is the force of the expanding gases on the bullet at the reference length l_1. Therefore, from the position of the maximum, the ratio of the reference force to the frictional force on the bullet can be determined. Second, the first root of (14) for $y = 0$, i.e., the smaller root of

$$a\left(1 - x_0^{1-\gamma}\right) - b(x_0 - 1) + 1 = 0 \tag{17}$$

gives the value of x_0, and hence the length of the barrel l_0 corresponding to the smaller intercept of the x–y graph with the l/l_1 axis in Fig. 1. This length (l_0), which we shall refer to as the “initial length” henceforth, is not necessarily the same as the actual length of the chamber, depending on the rate of deflagration of the solid propellant. This information, which can be obtained by fitting (14) to the experimental data, as we shall see in the next section, is important in understanding the thermodynamics of the barrel interior during the bullet or projectile propulsion.

4. Comparison with experimental data

Figures 2–5 show the function given in (14) and the experimental data that resulted from the four samples of 0.22-caliber rimfire ammunition discussed earlier. In each case we have used $\gamma = 1.235$ and the parameters a and b are fitted to the experimental data by a nonlinear least-squares analysis. For each ammunition, the shortest length of the barrel and the corresponding muzzle speed are taken to be l_1 and v_1, respectively. The values of the parameters a and b, along with the calculated values of x_0, F_1/f, l_0, and $l_{\max}$ are listed in Table 2 for the four ammunition types studied.

Fig. 2. Experimental (markers) and theoretical (solid line) relationship between muzzle speed and barrel length for the 0.22 rimfire CCI Stinger ammunition.

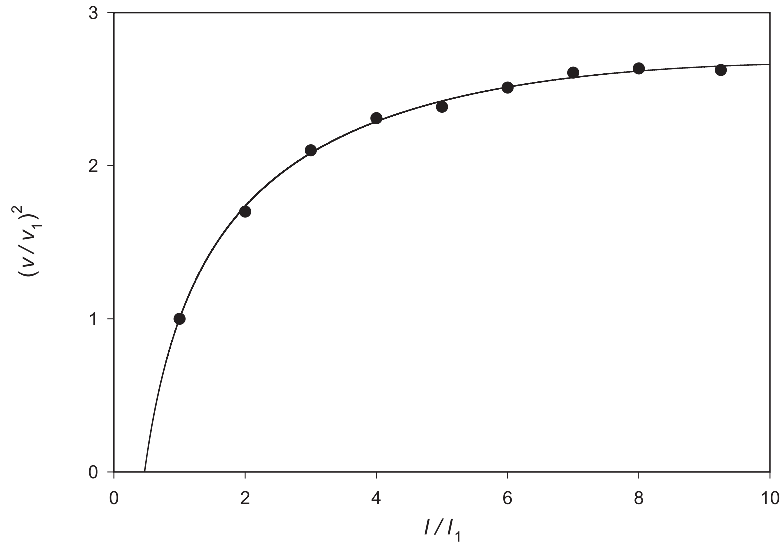


Fig. 3. Same legend as in Fig. 2 but with the WIN Wildcat ammunition.

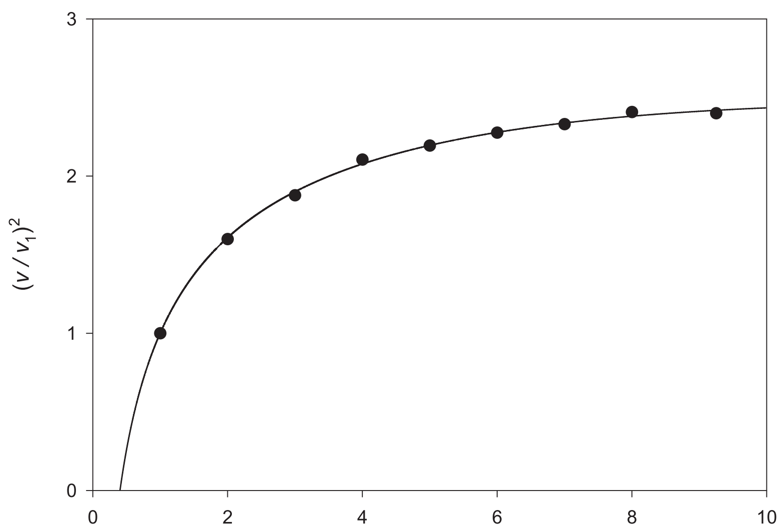


Table 2. Dimensionless parameters a and b of (14) obtained from nonlinear least-squares fits to the experimental data for different ammunition. The other physical quantities, as described in the article, are obtained from these parameters and the experimental data. In each case, l_{max} is the calculated optimal length of the barrel.

Ammunition	a	b	x_0	F_1 / f	l_0 (in.)	l_{max} (in.)
CCI Stinger	5.293	0.06106	0.4685	20.37	0.94	23.0
WIN Wildcat	4.319	0.04119	0.4042	24.64	0.81	26.8
REM HV	4.369	0.06268	0.4039	16.38	0.81	19.2
FED UM*	1.464	0.08199	0.09615	4.195	0.58	19.2

*UltraMatch (1000A).

Fig. 4. Same legend as in Fig. 2 but with the REM HV ammunition.

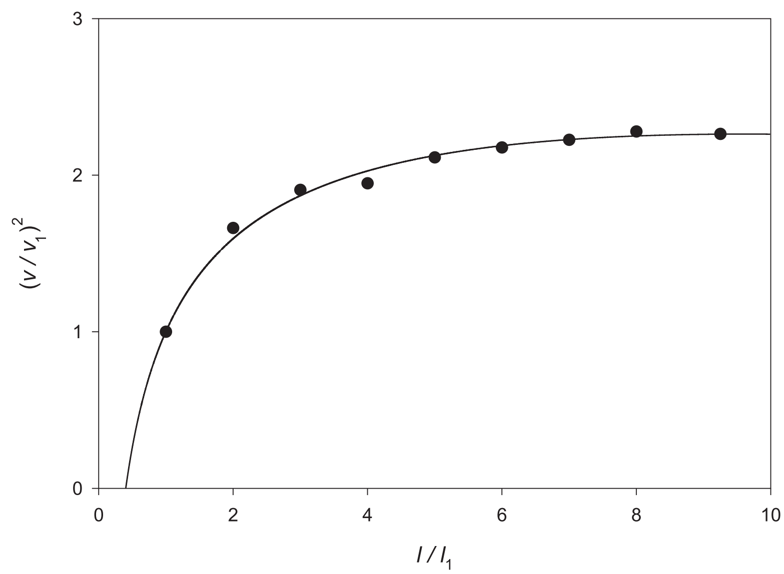
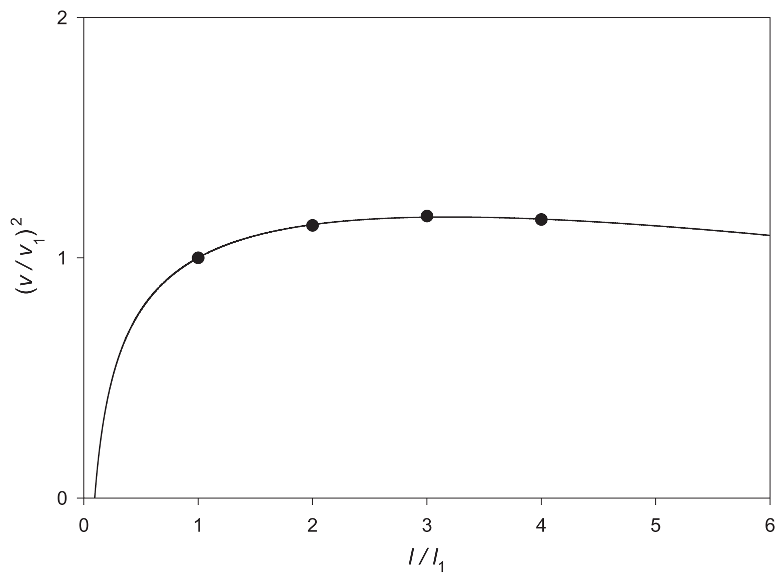


Fig. 5. Same legend as in Fig. 2 but with the FED UltraMatch (1000A) ammunition.



A point worth mentioning here is that by combining (9) and (16) it is straightforward to show that the optimal length of the barrel l_{max} is independent of the choice of the reference values for l_1 and v_1. The same is true for the initial length l_0, although in this case the result is less trivial because of the functional form of (17). Consequently, one can use any other pair of the experimental data for l_1 and v_1. This simply renormalizes the reduced quantities l/l_1 and v/v_1 such that they both become unity for that particular pair of data and clings the least-squares curve to that point. Accordingly, the values of a, b, x_{max}, and x_0 will all change but the optimal length l_{max} and the initial length l_0 remain the same within the statistical fluctuations of the experimental data. To demonstrate this, we used each and every pair of the experimental data for l_1 and v_1 for the four types of ammunition studied and calculated the average

Table 3. Mean values of the optimal length and the initial length, calculated by averaging over the values obtained using different pairs of the experimental data for l_1 and v_1 for each ammunition.

Ammunition	$< l_{max} >$ (in.)	$< l_0 >$ (in.)
CCI Stinger	23.2 ± 2.4	0.95 ± 0.03
WIN Wildcat	27.0 ± 3.2	0.82 ± 0.02
REM HV	22.1 ± 7.7	0.75 ± 0.05
FED UM*	19.2 ± 0.3	0.59 ± 0.03

*UltraMatch (1000A).

values of the resulting optimal lengths and initial lengths in each case. The results, which are shown in Table 3, are in good agreement with the values in Table 2 except for the REM HV ammunition. In this case, the discrepancy is due to relatively large statistical fluctuations of some of the experimental data as can be seen in Fig. 4. For instance, if we exclude the fourth point from the averaging process, we obtain a value of 19.6 ± 1.9 for $< l_{max} >$ and a value of 0.77 ± 0.02 for $< l_0 >$.

As can be seen from Figs. 2–5, the agreement between the functional form of (14) and the experimental data on muzzle speed as a function of barrel length is excellent. This is in contrast to the currently used analytical model, which assumes an average pressure in the barrel throughout the entire propulsion of the projectile [16]. Based on this model, and using the impulse–momentum relationship, the muzzle speed of the projectile is calculated to be

$$v = \frac{\bar{p}At}{m} \tag{18}$$

where $\bar{p}$ is the average pressure in the barrel, A is the cross-sectional area of the barrel interior, t is the time of propulsion in the barrel, and m is the projectile mass. Since $v = \mathrm{d}x/\mathrm{d}t$, (18) can be integrated and, after eliminating t by using (18) itself, we obtain

$$v^2 = \frac{\bar{p}A}{m}(l - l_1) \tag{19}$$

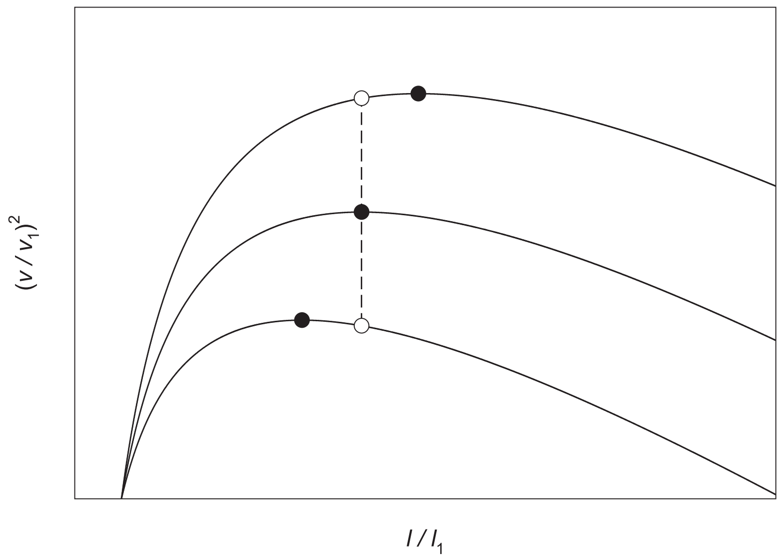
which results in a linear relationship between y and x (as defined earlier). Although a linear relationship is approximately true for small variations of the barrel length, it does not hold in general, as can be seen from the experimental data.

5. Optimal propellant mass

Let us now look at the scenario from a different perspective by posing the following question: For a given barrel length, how much solid charge or propellant is needed such that the given barrel length becomes optimum? To this end, we shall consider the given barrel length as an input. Then, experimentally measuring the frictional force f of the bore on the bullet or projectile, allows the reference force F_1 and hence the reference pressure p_1 to be calculated. This pressure, along with the known chemical composition of the propellant as well as the enthalpies of the deflagration reactions, can be used to estimate the ideal mass of the solid propellant required to render the given barrel length optimum.

Let us see what happens if we now increase or decrease the propellant mass from its ideal value. As the charge increases, the pressure p_1 and the velocity v_1 increase, which in turn change the values of the parameters a and b. Furthermore, according to (16), the ratio a/b and x_{max} both increase. Consequently, the representative point on the x–y diagram, corresponding to the given barrel length, moves to a higher graph but to the left of its maximum, as shown schematically in Fig. 6, making the barrel length shorter

Fig. 6. A schematic diagram showing the effect of the variation of propellant mass from its ideal value for which the barrel length is optimum (middle curve). The solid markers show the position of the maximum on each curve. The hollow markers indicate the new positions of the representative point on adjacent curves resulting from variations of the propellant mass from its ideal value at constant barrel length.



than the optimum length for the amount of charge present. Similarly, if the solid charge is decreased below its ideal value, the point moves to a lower graph but to the right of its maximum, making the barrel length longer than the optimum length for the amount of charge present.

6. Discussion and concluding remarks

The results of this work show that adiabatic expansion of gases in a rifle barrel leads to the correct functional dependence of the muzzle speed of a bullet or projectile on the barrel length. The currently used constant-pressure assumption during the entire propulsion, on the other hand, results in a linear relationship that, although correct for small barrel length variations, does not hold in general.

By fitting the two parameters of (14) to the experimental data and from (16) and (17), one can obtain interesting information on the details of the thermodynamics of the projectile propulsion inside the barrel. This includes the ratio of the force of the expanding gases at the reference point to the frictional force on the projectile, the initial length of the chamber l_0, and the optimal length of the barrel $l_{\max}$ corresponding to the maximum muzzle speed. Comparison of this information with the experimentally known or measurable data such as the chamber length, the optimum barrel length, and the frictional force of the bore on the projectile can shed light on the details of the process.

Finally, in our treatment we have ignored the rotational kinetic energy carried by the spinning bullet as a result of the rifling of the barrel. The justification is that this energy is much smaller compared to the translational kinetic energy of the bullet. Indeed, the former is only of the order of 0.25–1% of the total available energy [17].

To the best of our knowledge, no theoretical analysis of this type has been carried out in the literature.

Acknowledgment

We are grateful to Dr. A. Lynn Anderson for many helpful discussions and suggesting useful references. We also thank Kim M. Dalebroux for editing and critiquing the final manuscript.

References

1. M. White. The quiet rimfire and more thoughts on an accurate 10/22. Precision Shooting Magazine, Special 3, **2**, November 1995.
2. S.T. Kunz. 0.22 rimfire barrel length versus velocity. World Wide Web, at http://members.home.net/stkunz/stkunz/bblvsvel.htm. 25 January 2001.
3. The New Encyclopedia Britannica. 15th ed. The University of Chicago, Chicago. 1993. Vol. 2, p. 206.
4. V. Lindner. *In* Kirk-Othmer encyclopedia of chemical technology. 3rd ed. *Edited by* H.F. Mark et al. Wiley, New York. 1980. Vol. 9, p. 620.
5. L.F. Fieser and M. Fieser. Advanced organic chemistry. Reinhold, New York. 1961. p. 298.
6. The New Encyclopedia Britannica. 15th ed. The University of Chicago, Chicago. 1993. Vol. 5, p. 571.
7. R.A. Rinker. Understanding firearms ballistics. 4th ed. Melberry House Publishing, Apache Junction, Ariz. 2000. pp. 28–29.
8. V. Lindner. *In* Kirk-Othmer encyclopedia of chemical technology. 3rd ed. *Edited by* H.F. Mark et al. Wiley, New York. 1980. Vol. 9. p. 625.
9. D. Halliday, R. Resnick, and J. Walker. Fundamentals of physics. 5th ed. Wiley, New York. 1997. pp. 499.
10. H.B. Callen. Thermodynamics. Wiley, New York. 1960. p. 326.
11. P.W. Atkins. Physical chemistry. 3rd ed. W.H. Freeman and Company, New York. 1986. pp. 541 and 454–455.
12. P.W. Atkins. Physical chemistry. 3rd ed. W.H. Freeman and Company, New York. 1986. pp. 532–533.
13. V. Lindner. *In* Kirk-Othmer encyclopedia of chemical technology. 3rd ed. *Edited by* H.F. Mark et al. Wiley, New York. 1980. Vol. 9, p. 627.
14. V. Lindner. *In* Kirk-Othmer encyclopedia of chemical technology. 3rd ed. *Edited by* H.F. Mark et al. Wiley, New York. 1980. Vol. 9, p. 630.
15. R.A. Rinker. Understanding firearms ballistics. 4th ed. Melberry House Publishing, Apache Junction, Ariz. 2000. pp. 54 and 118.
16. R.A. Rinker. Understanding firearms ballistics. 4th ed. Melberry House Publishing, Apache Junction, Ariz. 2000. p. 39.
17. R.A. Rinker. Understanding firearms ballistics. 4th ed. Melberry House Publishing, Apache Junction, Ariz. 2000. p. 140.